Henk Druiven, August 27, 2019.

Calculating with relativistic quantities can be a lot easier.

By introducing a single new definition and using a different postulate than that of Einstein (Wikipedia), relativistic calculations can be made a lot easier.

The special theory of relativity was developed by Albert Einstein in 1905. This theory is based on the following two postulates:

> The laws of physics take the same form in all inertial frames of reference.
>
> As measured in any inertial frame of reference, light is always propagated in empty space with a definite velocity c that is independent of the state of motion of the emitting body. Or: the speed of light in free space has the same value c in all inertial frames of reference.

I would like to change the second postulate to the following:

> Time and space are indistinguishable. Time does not exist alongside our three known dimensions, but is equal to that.
>
> Time relates to distance as 1 second to 299,792,458 m. It does not matter in which direction this distance is bridged.

From the postulates of Einstein it follows that:

$$t' = t\sqrt{1-\frac{v^2}{c^2}}$$

In other words; "If an object moves at a constant speed relative to a non-accelerating observer, the observer's time is measured as t' ".

This formula can be rewritten as:

$t'^2 = t^2 - \frac{s^2}{c^2}$ or $t^2 = t'^2 + \frac{s^2}{c^2} = t'^2 + s_c^2$. s_c is the distance expressed in light seconds.

Hence my second postulate.

I would like to define a second definition for the speed of an object.

The normal definition of velocity is: The velocity (v_i) of an object relative to an observer is the distance traveled in the inertial system of the observer by that object divided by the time (***t***) that according to the observer that object requires to bridge that distance .

The new definition is: The velocity (v_e) of an object relative to an observer is the distance traveled in the inertial system of the observer by that object divided by the time (***t'***) of the object itself.

We can now derive the following formulas:

=> $t^2 = t'^2 + s_c^2$

=> $\frac{t^2}{s_c^2 . c^2} = \frac{1}{c^2} + \frac{t'^2}{s_c^2 . c^2}$

=> $\frac{1}{v_i^2} = \frac{1}{c^2} + \frac{1}{v_e^2}$ v_i is the speed of the object according to the observer
v_e is the speed of the object according to the new definition

=> $1/v_e^2 = 1/v_i^2 - 1/c^2$ => $v_e = \frac{1}{\sqrt{1/v_i^2 - 1/c^2}}$ or $v_e = \frac{v_i}{\sqrt{1 - v_i^2/c^2}}$ (A)

=> $1/v_i^2 = 1/v_e^2 + 1/c^2$ => $v_i = \frac{1}{\sqrt{1/v_e^2 + 1/c^2}}$ or $v_i = \frac{v_e}{\sqrt{1 + v_e^2/c^2}}$ (B)

=> $v_i^2 = \frac{v_e^2 c^2}{c^2 + v_e^2}$ => $v_i^2/c^2 = \frac{v_e^2/c^2}{1 + v_e^2/c^2}$

=> $1 - v_i^2/c^2 = \frac{1 + v_e^2/c^2}{1 + v_e^2/c^2} - \frac{v_e^2/c^2}{1 + v_e^2/c^2}$

=> $\sqrt{1 - v_i^2/c^2} = \frac{1}{\sqrt{1 + v_e^2/c^2}}$ of $\sqrt{1 - v_i^2/c^2} . \sqrt{1 + v_e^2/c^2} = 1$ (C)

(A) is useful if you want v_i to replace with v_e .
(B) if you want v_e to replace with v_i .
(C) if you wantt $\sqrt{1 - v_i^2/c^2}$ o replace with $\frac{1}{\sqrt{1 + v_e^2/c^2}}$ or vice versa.

Now some calculation examples:

Example 1

Suppose we shoot a clock at an enormous speed to Mars and back again at the same speed. Mars is at that moment 10 light minutes = 10 x 60 x 300,000 km = 180 million km away from us (360 million km vice versa, 20 light minutes).

According to an observer on earth, the clock takes 40 minutes to return. What is the clock at if it is set to 0:00:00 on departure?

Answer:

$$t_{clock} = t' = \sqrt{t^2 - s_c^2} = \sqrt{40^2 - 20^2} = 34.64\, min = 34:38\, min.$$

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Example 2

According to an observer, two rockets fly towards each other. According to the observer, rocket 1 is flying at a speed of 0.6 c, Rocket 2 is flying at a speed of 0.4 c. How fast do the rockets see each other approaching?

Answer:

For Rocket 1:

$$v_e = \frac{c}{\sqrt{1/0.6^2 - 1}} = 0.75\,c$$

For Rocket 2:

$$v_e = \frac{c}{\sqrt{1/0.4^2 - 1}} = 0.4363\,c$$

Sum of both = 1,1864 c

$$v_i = \frac{c}{\sqrt{(c/v_e)^2 + 1}} = \frac{c}{\sqrt{1/1.1864^2 + 1}} = 0.7646\,c$$

or the two rockets see each other approaching at a speed of 0.7646 c.

Example 3:

What is the kinetic energy of a particle with a speed of 0.99 c?

Answer:

$$v_e = \frac{c}{\sqrt{1/99^2 - 1}} = 7.018\,c$$

$$E_k = mc\sqrt{c^2 + v_e^2} - mc^2$$

$$E_k = mc^2\sqrt{1 + 7.018^2} - mc^2 = 6.089\,mc^2$$

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Example 4:

In the Large Hadron Collider (LHC) near Geneva, protons are accelerated in a ring-shaped tunnel with a circumference of 27 km to a speed of 0.999,999,964 c.

What is the required average field strength of the magnetic field to keep the protons in their orbit?

Answer:

An electrically charged particle traverses a (part of a) circular orbit in a magnetic field where the radius is determined by $r=\frac{mv_e}{Bq}$

=> $B=\frac{mv_e}{rq}$

m = the mass of the proton = $1.672{,}62*10^{-27}kg$
q = the charge of the proton = $1.6*10^{-19}C$
r = the radius of the completed circle = $4{,}297.2\,m$
v$_i$ = the speed of the proton = $0.999{,}999{,}964\,c$

$$v_e=\frac{c}{\sqrt{(c/v_i)^2-1}}=3{,}726.78\,c$$

$$B=\frac{1.672{,}62*10^{-27}*3.726{,}78*10^3*3*10^8}{4.297{,}2*10^3*1.6*10^{-19}}=2.72\,T$$

Example 5:

How large is the centrifugal force on a proton in the Large Hadron Collider (LHC) that must be compensated by the Lorentz force to keep the proton in its orbit. The LHC has a circumference of 27 km and the proton has a speed of 0.999.999.964 c.

Answer:

The outwardly directed force exerting a rotating mass m at a distance **r** from a center point is given by:

$$F_m=\frac{mv_e^2}{r}$$

m = the mass of the proton = $1.672{,}62*10^{-27}kg$
r = the radius of the completed circle = $4{,}297.2\,m$
v$_i$ = the speed of the proton = $0.999{,}999{,}964\,c$

$$v_e = \frac{c}{\sqrt{(c/v_i)^2 - 1}} = 3{,}726.78\,c$$

$$F_m = \frac{1.672{,}62 * 10^{-27} * (3{,}726.78\,c)^2}{4{,}297.2} = 4.865{,}4 * 10^{-7}\,N$$

Example 6:

With a spaceship, a clock with a constant acceleration is fired into space. An observer stays behind.

The clock is set at 00:00:00 at the time of departure.

The acceleration is $1000\,m/s^2$.

According to the observer, at what distance from the observer does the ship reach a speed of 0.4 c and what shows the clock at that moment?

Answer:

$$v_e = \frac{c}{\sqrt{(c/v_i)^2 - 1}} = 0.4364\,c$$

t' is what the clock shows.

$$t' = \frac{v_e}{a} = \frac{0.4364\,c}{1000} = 130{,}931\,seconds = 36:22:11$$

$$s = \frac{v_e * t'}{2} = \frac{a * t'^2}{2} = \frac{1000 * 130{,}930^2}{2} = 8.5714 * 10^{12}\,m = 8.5714 * 10^9\,km = 28{,}571\,light\ seconds$$

which is almost equal to $8\,light\,hour$.

Example 7:

After a hundred years, how much is a clock ahead on a 100-meter-high tower compared to a clock on the ground?

Answer:

The acceleration on the surface of the earth (and at a hundred meters height) is $10\,m/s^2$, which equals a time gradient per meter of $10\,m/(3 * 10^8\,m)^2 = 1.1111 * 10^{-16}\,per\ meter$ (one second equals $3 * 10^8\,m$) .

100 years is equal to $100*365*24*3600\,s=3.1536*10^{9}\,s$
The time difference is
$$height*time\,gradient*100\,years=100*1.1111*10^{-16}*3.1536*10^{9}=35.04*10^{-6}\,s$$
so just over $35\,\mu s$.

Example 8:

How much does a clock run relative to a clock on Earth and a satellite at a height of 20,000 km after one revolution?

Answer:

A clock at a certain height runs faster then a clock on earth.

A clock that moves relative to a clock on Earth is lagging behind.

The following applies to the acceleration due to the attraction of a mass:
$$g(r)=\frac{GM}{r^2}$$
The product of **G** and **M** is known to the earth with great accuracy.

For the earth applies: $GM=3.986*10^{14}\,m^3 s^{-2}$

For the satellite, the centrifugal force is equal to the attraction of the earth:

$$\frac{mv_e^2}{R_s}=\frac{mGM}{R_s^2} \quad => \quad v_e=\sqrt{\frac{GM}{R_s}}=\sqrt{\frac{3.986*10^{14}}{26.371*10^{6}}}=3{,}887.8\,ms^{-1}$$
The circumference of this circular orbit: $C_s=2\pi r=2*3.1416*26.371*10^{6}=165.694*10^{6}\,m$

The time of a revolution: $t'=\frac{165.694*10^{6}}{3{,}887.8}=42{,}619\,s=11:50:19$

The time difference due to its speed between the clock on Earth and the clock in the satellite is: $t-t'=\frac{s}{v_i}-\frac{s}{v_e}=\frac{s(v_e-v_i)}{v_i v_e}=\frac{s(1-v_i/v_e)}{v_i}$

$$v_e=\frac{v_i}{\sqrt{1-v_i^2/c^2}} \quad => \quad t-t'=\frac{s}{v_i}\left(1-\sqrt{1-\frac{v_i^2}{c^2}}\right)\approx\frac{s\,v_i}{2c^2}$$
The time difference due to its speed after one revolution:
$$t-t'=\frac{165.694*10^{6}*3{,}887.8}{2(3*10^{8})^2}=3.5788*10^{-6}\,s$$

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If you do not like approximations, calculate the result by using Google's Ivy calculator:

$$t-t'=\frac{s}{v_i}(1-\sqrt{1-\frac{v_i^2}{c^2}})=3.5788*10^{-6}s$$

In Example 7, g is approximately a constant. However, that does not apply here. We therefore integrate **g(r)** from R_0 to R_s.

$$g[R_0-R_s]=\int_{R_0}^{R_s}\frac{GM}{r^2}dr=[\frac{GM}{r}]_{R_o}^{R_s}=GM(-\frac{1}{R_s}+\frac{1}{R_0})=GM(\frac{R_0-R_s}{R_s R_0})m^2 s^{-2}$$

$$g[R_0-R_s]=3.986*10^{14}(\frac{6.371*10^6-26.371*10^6}{6.371*10^6*26.371*10^6})=-47.450*10^6 m^2 s^{-2}$$

The time difference of the two clocks due to gravity after one revolution is:

$$\frac{-47.45*10^6}{(3*10^8)^2}*42,619=-22.47*10^{-6}s$$

The total time difference after one revolution is therefore

$3.5788*10^{-6}-22.47*10^{-6}=-18.89*10^{-6}s$, so almost $-19\,\mu s$

Also see:

The special theory of relativity for dummies.

Can anything other than the expansion of the universe explain the redshift?